Quantum chaos in topologically massive gravity

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based on: 2005.08508 (with Avinash Raju)

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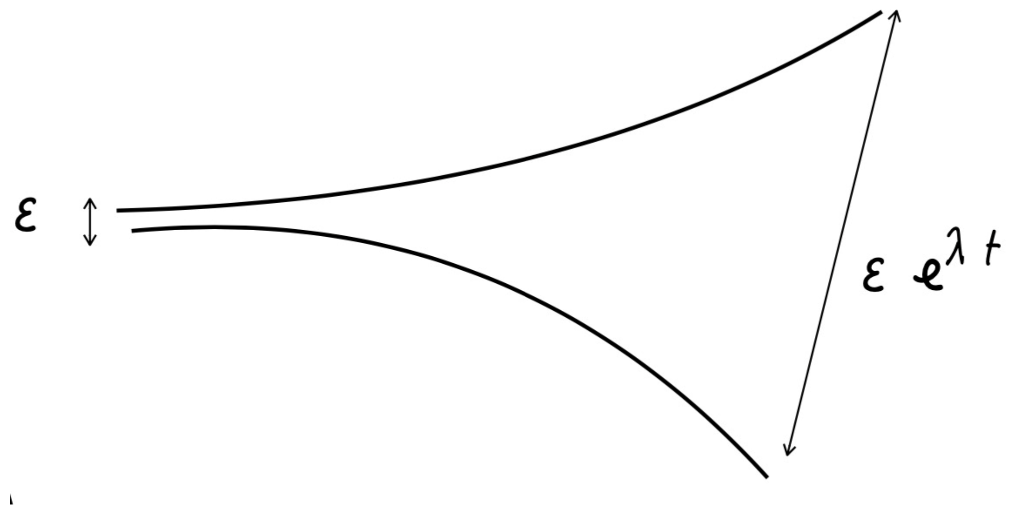
Classical Chaos

- Early time: exponential sensitivity of phase-space trajectories to the initial conditions



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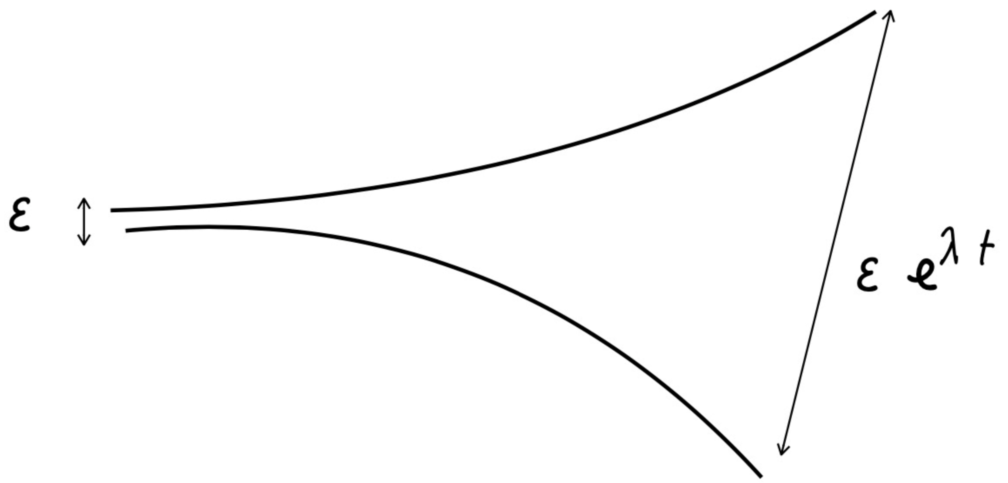
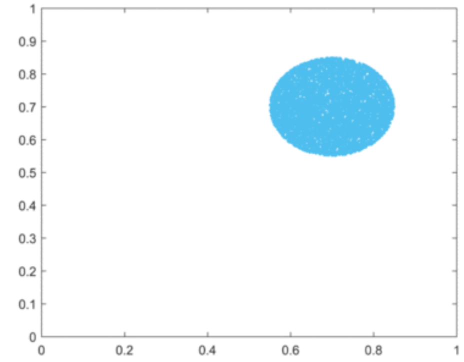


- Long time: mixing



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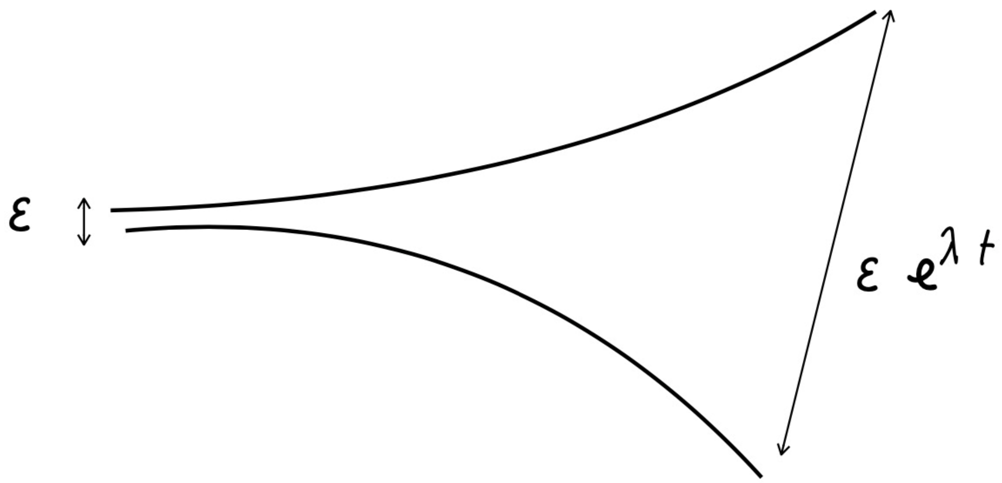

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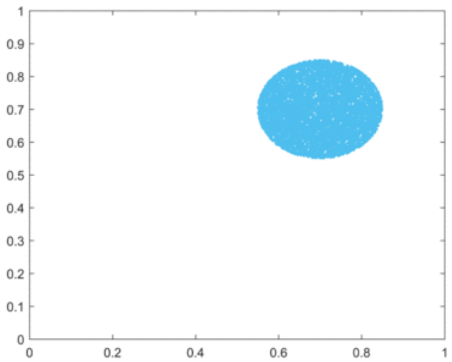
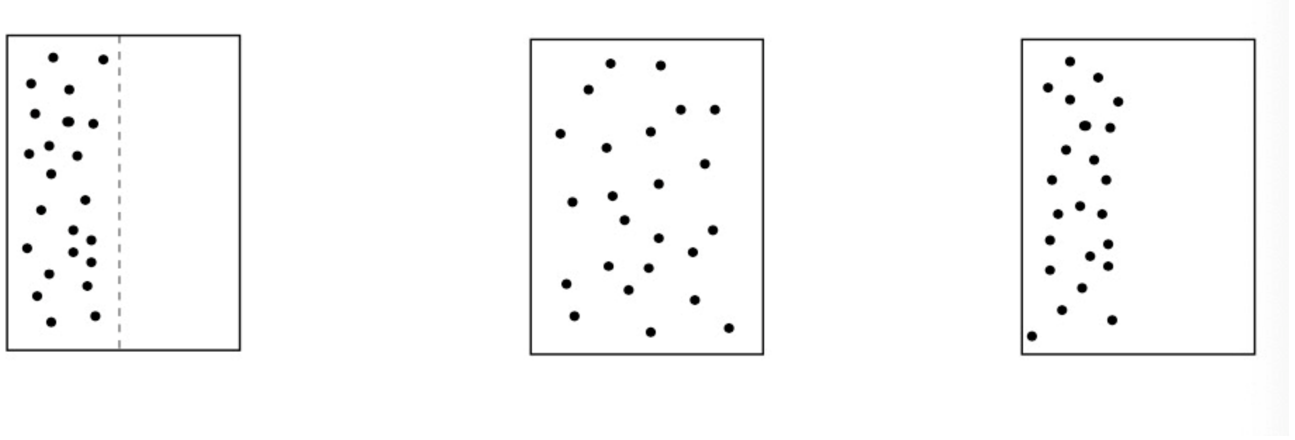


- Very late time: Poincare recurrences



Classical Chaos vs Quantum Mechanics

- In quantum systems, the 'classical' coarse-graining is set by $\hbar$

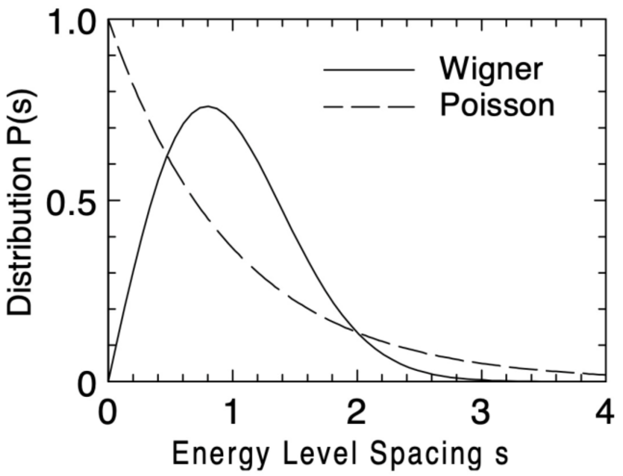
- The picture above might not be useful (after Ehrenfest time scale) $t_{\mathrm{Ehrenfest}} \sim \frac{1}{\lambda} \log\left(\int_{\Sigma} pdq/\hbar \right)$

- How the previous discussion should be modified due to QM

- Classical chaos for nonzero $\hbar$? Quantum chaos?

Quantum Chaos

- Quantize the classical chaotic system: chaotic systems have statistical level repulsion characteristic of random matrices [Review by D'Alessio, Kafri, Polkovikov, Rigol, 1509.06411]



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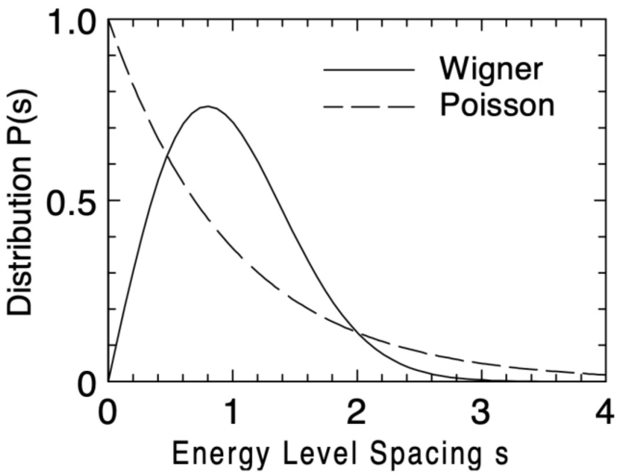


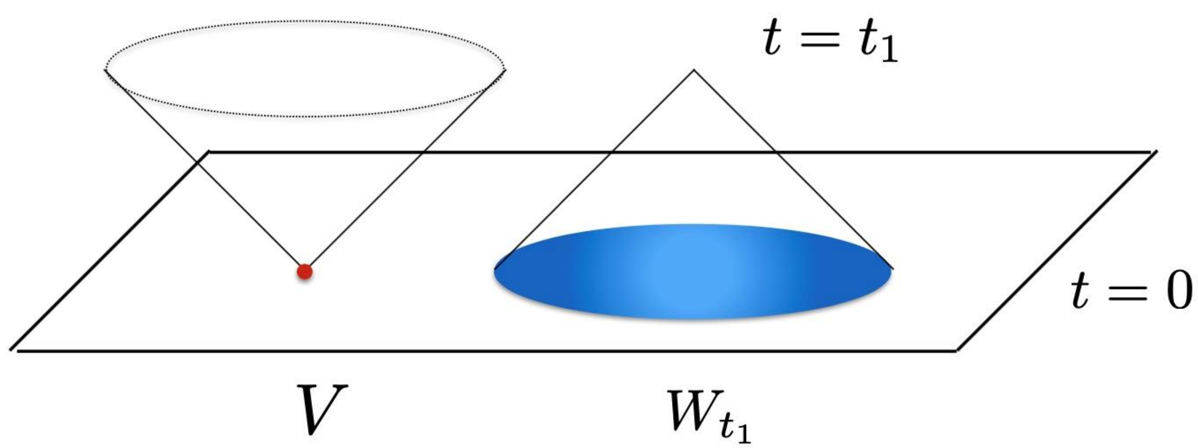
- Local chaotic behavior can be generalized on the short time scale (semi-classical intuition)

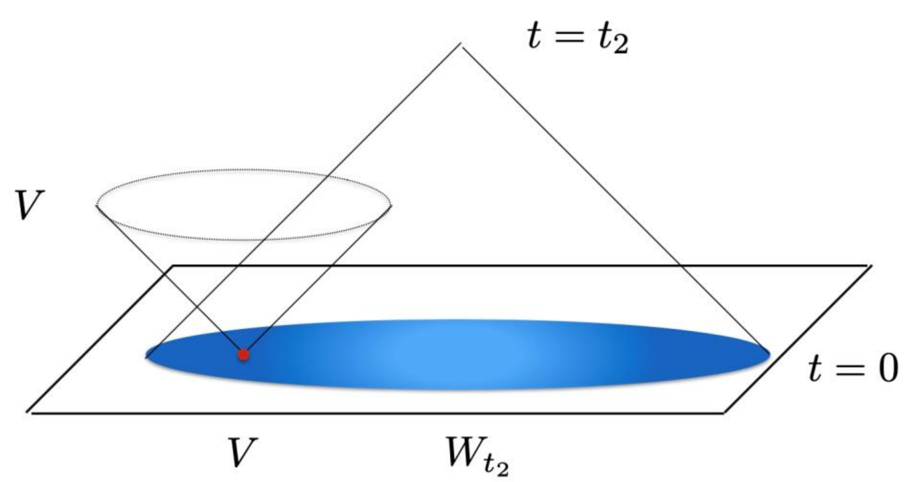
$$\frac{\partial q(t, p_0)}{\partial p_0} = \{q(t), p_0\}_{\mathrm{PB}} \quad \Rightarrow \quad \langle [V, W_t]^2 \rangle \sim e^{\lambda t}$$

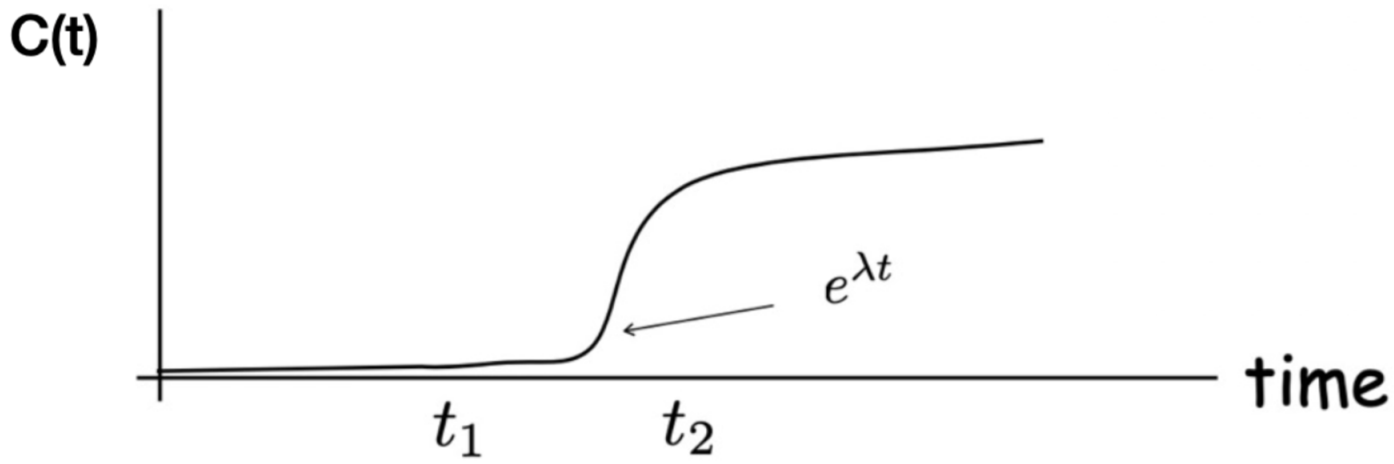
Expectation value of the commutators

[Larkin, Ovchinnikov, JETP (1969); Shenker, Stanford, 1306.0622; Maldcena, Shenker, Stanford, 1503.01409]

$$C(t, \mathbf{x}) = -\langle [W(t, \mathbf{x}), V(0)]^\dagger [W(t, \mathbf{x}), V(0)] \rangle_\beta$$







Out-of-Time-Ordered Correlators (OTOC)

[Larkin, Ovchinnikov, JETP (1969); Shenker, Stanford, 1306.0622; Maldcena, Shenker, Stanford, 1503.01409]

$$C(t,\mathbf{x}) = -\langle [W(t,\mathbf{x}), V(0)]^\dagger [W(t,\mathbf{x}), V(0)] \rangle_\beta$$

- Probing the sensitivity of a system to the initial conditions

$$C_2 = C_1 - C(t,\mathbf{x})$$

$$C_2 = \langle \Psi_1(t)|\Psi_2(t)\rangle + \langle \Psi_2(t)|\Psi_1(t)\rangle$$ → **Out-of-time Ordered**

$$C_1 = \langle \Psi_1(t)|\Psi_1(t)\rangle + \langle \Psi_2(t)|\Psi_2(t)\rangle$$ → **Time ordered**

$$|\Psi_1(t)\rangle = V_0 W_t |\mathrm{TFD}\rangle$$

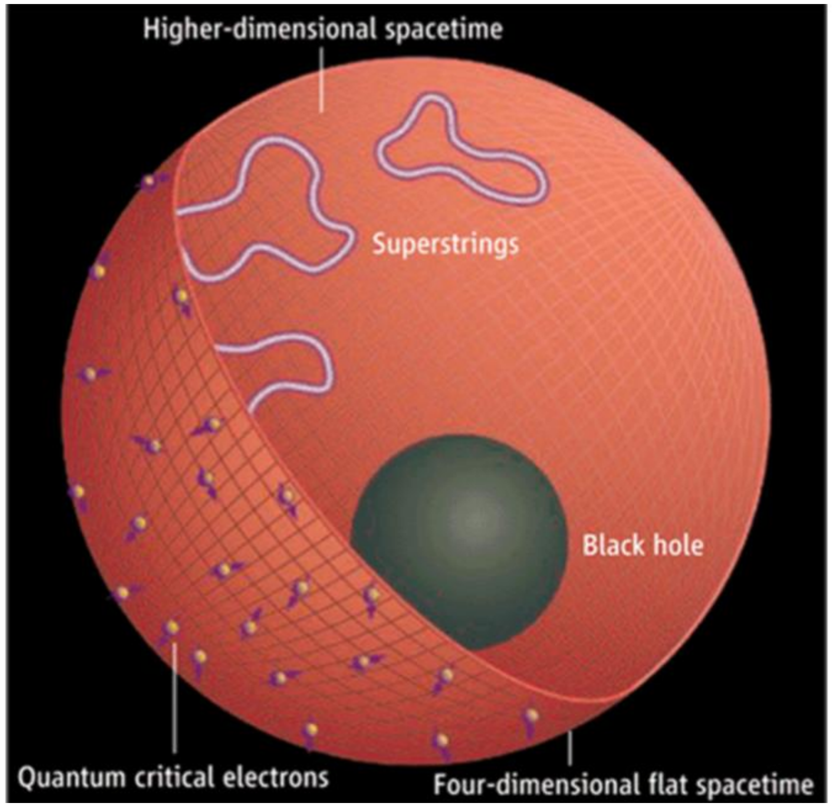
$$|\Psi_2(t)\rangle = W_t V_0 |\mathrm{TFD}\rangle$$

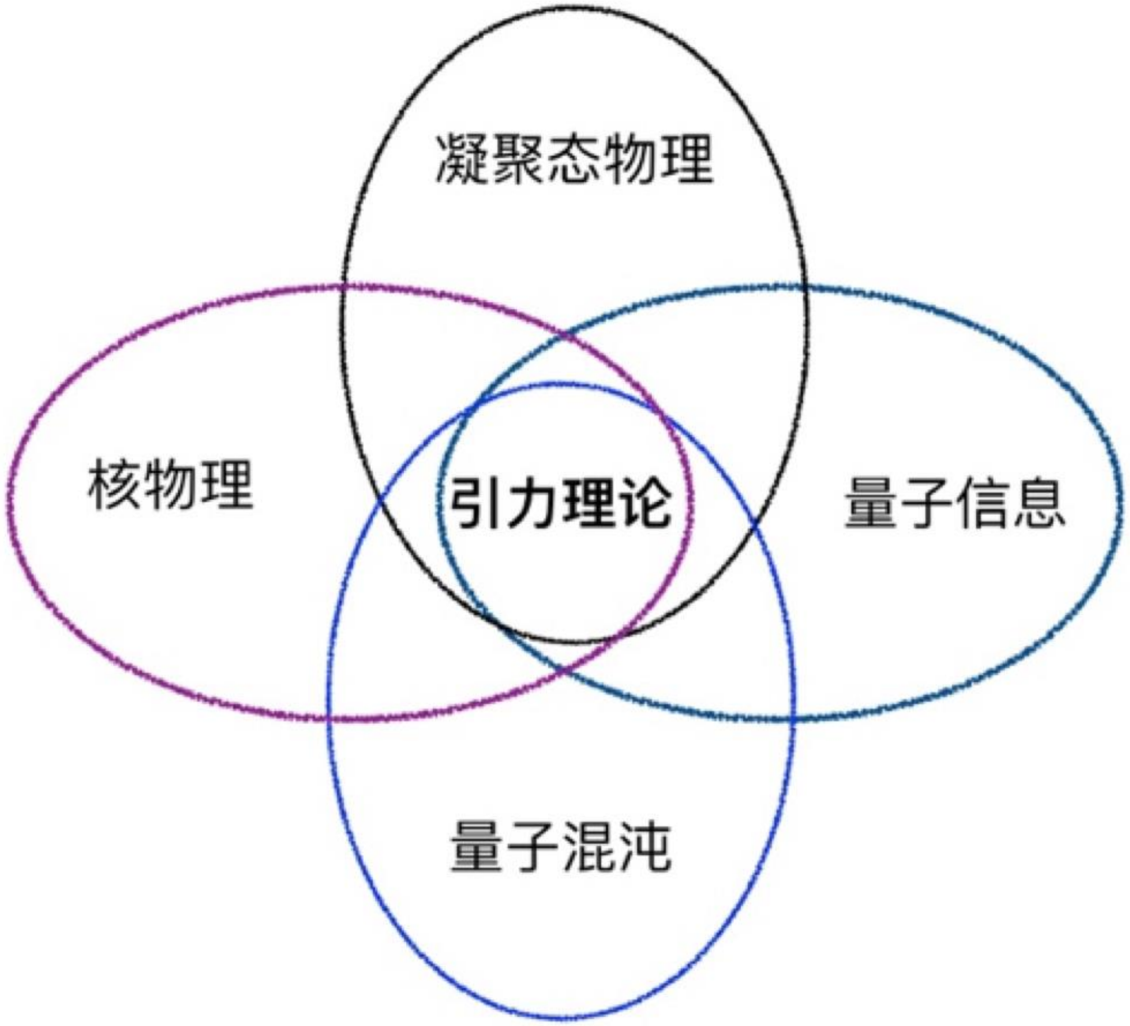
- Lyapunov exponents, butterfly velocities: for interacting quantum systems with many degrees of freedom

$$C_2 = 1 - \epsilon e^{\lambda_L\left(t - \frac{x}{v_B}\right)} \qquad t_r \ll t \ll t_*$$

Different (diffusive) spreading might be seen in non-maximally chaotic systems

Holographic duality





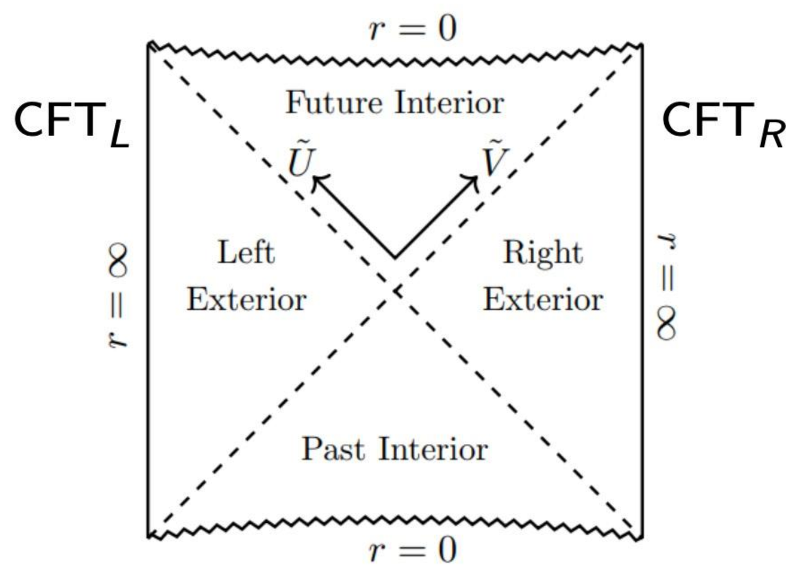
Holographic OTOC

[Shenker, Stanford, 1306.0622; 1412.6087]

- OTOC

$$F = \langle TFD | V(t_1) W(t_2) V(t_3) W(t_4) | TFD \rangle = \langle out | in \rangle$$

$$|TFD\rangle =$$



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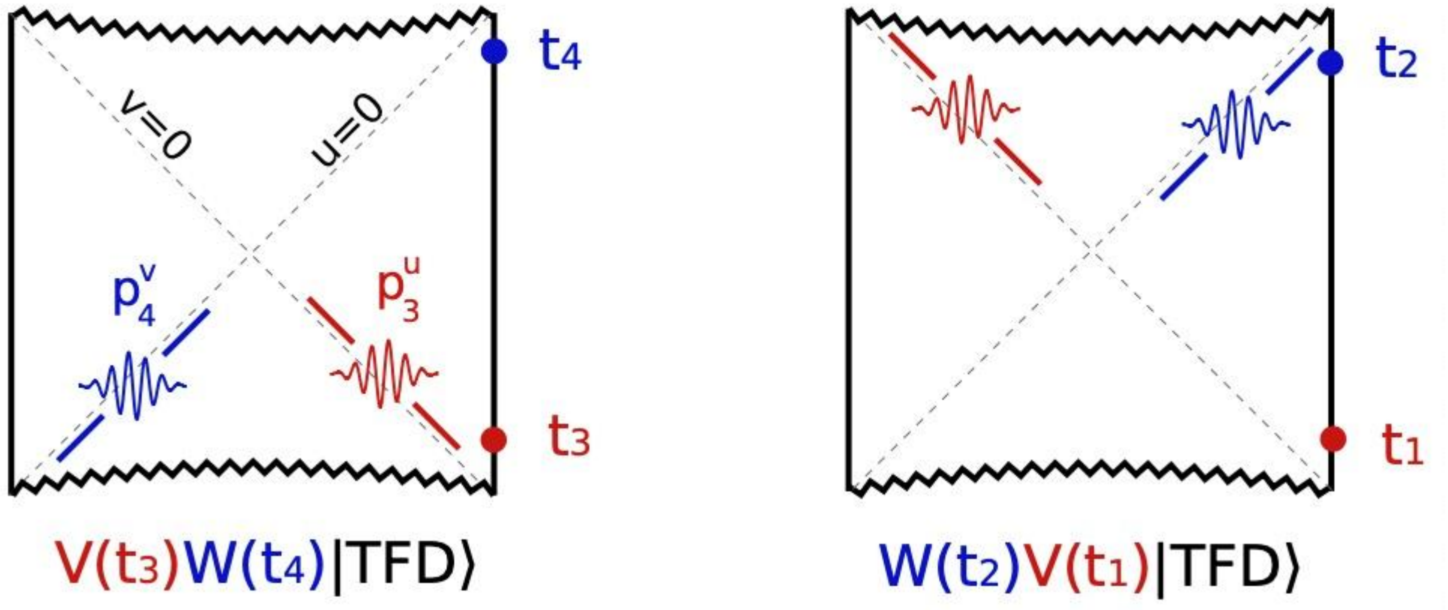

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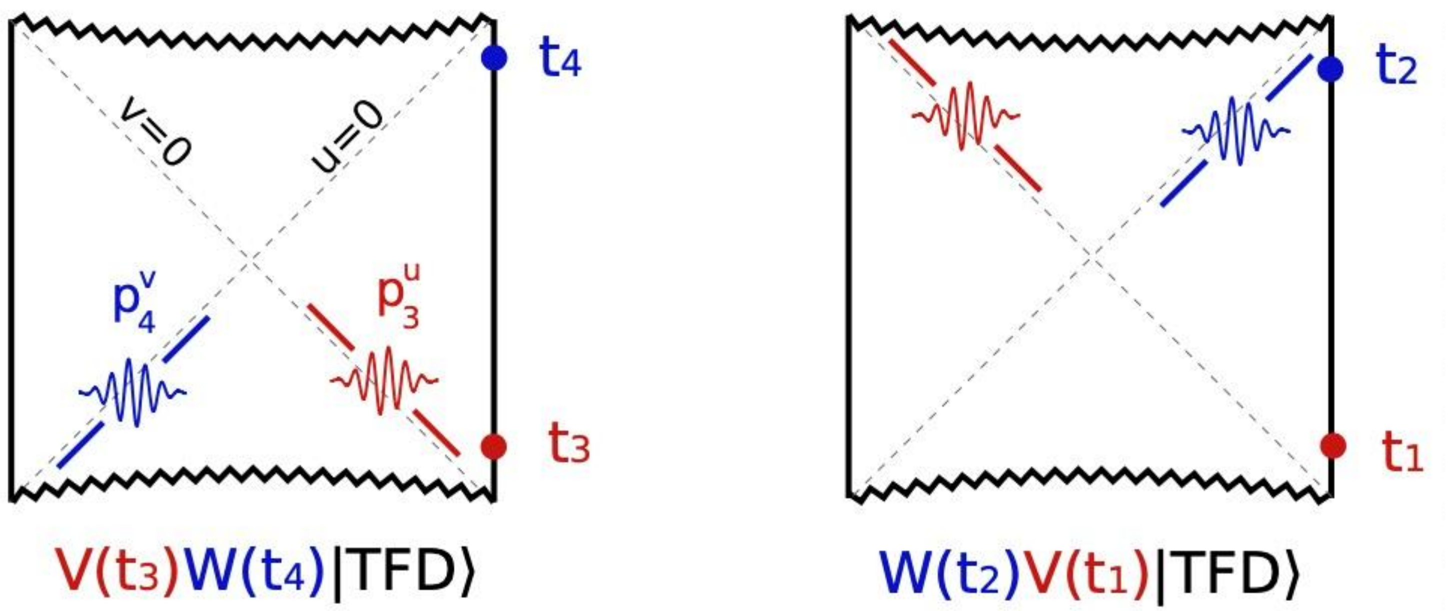
[Shenker, Stanford, 1306.0622; 1412.6087]

- OTOC = amplitudes for 2-to-2 scatterings of particles dual to W and V in a black hole geometry dual to the thermal state $|\mathrm{TFD}\rangle$



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- In elastic eikonal gravity approximation, the dominate contribution is related to the gravitational shock waves on the horizon of a two-sided black hole

$$ds^2_{shock} = d\bar{s}^2 + h_{uu}du^2 + h_{vv}dv^2$$

$$\mathrm{OTOC} = \int K_V K_W K_V K_W \times e^{i\delta(s,b)}$$

(Bulk 4-pt function under approximation)

$$\delta(s,b) = \frac{1}{4}\int d^{d+1}x\sqrt{-g}\,(h_{uu}T^{uu} + h_{vv}T^{vv}) \propto G_N s f(b) \sim G_N e^{\frac{2\pi}{\beta}\left(t - \frac{b}{v_B}\right)}$$

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- Universal Lyapunov exponent $\lambda_L = 2\pi T$

- The butterfly velocity depends on the details of the black hole geometry

Chaos bound [Maldacena, Shenker, Stanford, 1503.01409]

- Related regulated function

$$F(t) = \mathrm{Tr}\big(yV(0)yW(t)yV(0)yW(t)\big) \sim 1 - \epsilon e^{\lambda_L t}\,, \quad y^4 = \frac{e^{-\beta H}}{Z}$$

- For systems with large hierarchy between thermalization and scrambling, analyticity in correlation functions demands

$$\lambda_L \leq 2\pi T$$

- It holds for very generic quantum many-body systems

- Black holes saturate this bound: maximal chaos

- SYK/AdS2 [Kitaev, 2015]

Chaos from hydrodynamics via pole skipping

[Grozdanov, Schalm, Scopelliti, 1710.00921; Blake, Lee, Liu, 1801.00010; Blake, Davison, Grozdanov, Liu, 1809.01169; …]

- Naively hydrodynamics has nothing to do with chaos

- Deep connection from EFT: Signatures of chaos in energy density two point function of $G^R_{T^{00}T^{00}}(\omega, k)$

- There exists a special point $(\omega_*, k_*) = (i\lambda_L, \frac{i\lambda_L}{v_B})$

  in $G^R(\omega, k) = \frac{B(\omega, k)}{A(\omega, k)}$ with $A(\omega_*, k_*) = B(\omega_*, k_*) = 0$

- Examples of pole skipping in many maximally chaotic systems: SYK, AdS black holes in Einstein gravity plus matter

- Pole skipping also exists for 2-pt correlators of other operators on the lower half plane

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Motivation

- Connection between OTOC and pole skipping, e.g. for systems with multiple Lyapunov exponents or non-maximally chaotic systems
- What is the role of rotation in holographic chaos
- What is the role of massive graviton in holographic chaos

Why 3D gravity

- A “simple” toy model to understand quantum gravity
- We can learn much from CFT calculations
- In the following, we will talk about
  - Quantum chaos in 3D Einstein gravity
  - Quantum chaos in 3D TMG

3D Einstein gravity

- Einstein-Hilbert action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\,\sqrt{-g}\,(R-2\Lambda)$$

- BTZ Black hole solution

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\varphi - \frac{r_+ r_-}{\ell r^2}dt\right)^2,$$
$$f(r) = \frac{(r^2-r_+^2)(r^2-r_-^2)}{\ell^2 r^2}.$$

- M, T, Ω are determined by r_+, r_-.

- The dual theory is expected to be a CFT with $\beta_\pm = \beta(1\mp\ell\Omega)$ and
$$c_+ = c_- = \frac{3\ell}{2G}$$

- The angular direction is periodic. At high temperature $\frac{\beta}{\ell}\to 0$, we can take a ``decompactification'' limit (a boosted brane)

Chaos parameters from OTOC

[Jahnke, Kim, Yoon, 1903.09086; Stikonas 2018; Poojary 2018]

- From shock wave calculations

$$\mathrm{OTOC}(t,\varphi_{12}) \simeq 1 + \epsilon e^{\frac{2\pi}{\beta}t} h(\Omega t - \varphi) \simeq 1 + C_1 e^{\frac{2\pi}{\beta_+}(t+\ell\varphi_{12})} + C_2 e^{\frac{2\pi}{\beta_-}(t-\ell\varphi_{12})}$$

- Naively we have

$$\lambda_\pm = \frac{2\pi}{\beta(1\mp\Omega\ell)} \qquad\qquad v_\pm = \mp 1$$

- The chaos bound is violated: $\lambda_- < \frac{2\pi}{\beta} < \lambda_+$

- However, the angular coordinate is periodic, i.e. the profile of shock wave is periodic, therefore the two coefficients C_1 and C_2 are not independent [Mezei, Sarosi, 1908.03574]

$$\mathrm{OTOC}(t,\varphi_{12}) \simeq 1 + \epsilon\left[e^{\frac{2\pi}{\beta_+}(t+\ell\varphi_{12})} + \# e^{\frac{2\pi}{\beta_-}(t-\ell\varphi_{12})}\right]$$

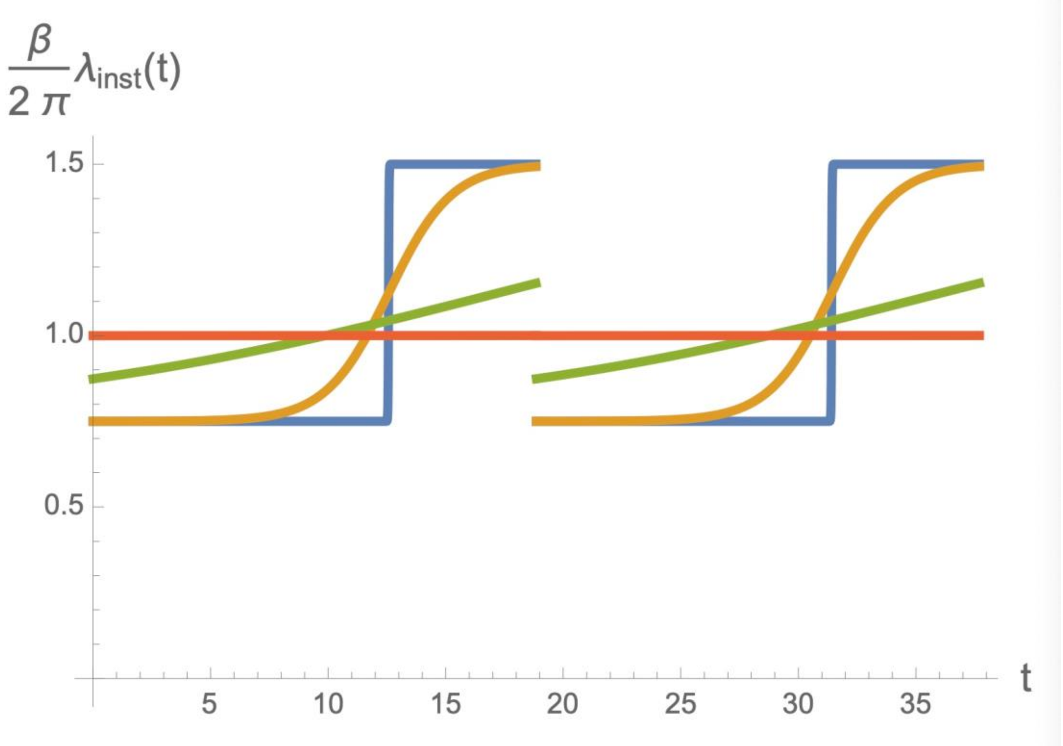
Instantaneous Lyapunov exponent

[Mezei, Sarosi, 1908.03574]

$$\mathrm{OTOC}(t, \varphi_{12}) \simeq 1 + \epsilon \left[e^{\frac{2\pi}{\beta_+}(t+\ell\varphi_{12})} + \# e^{\frac{2\pi}{\beta_-}(t-\ell\varphi_{12})} \right]$$

- instantaneous Lyapunov exponent

$$\mathrm{OTOC}(t, 0) \simeq 1 + \epsilon e^{\lambda_{\mathrm{inst.}} t}$$



$$\beta = 0, 2\pi, 16\pi, \infty$$

- In the high temperature limit, the instantaneous Lyapunov exponents behave as step function.

- The average of instantaneous Lyapunov exponents is $\frac{2\pi}{\beta}$

Pole skipping in holography

- From EOM near the horizon $E_{vv} = 0$

  Expand $h_{ab} = e^{-i\omega v + ik\phi}(r-r_+)^\gamma \sum_{n=0}^{\infty} \tilde{h}_{ab}^{(n)}(r-r_+)^n$

  near horizon,

$$\left(2\pi i\omega + 4\pi i\Omega k - k^2\beta(1-\Omega^2)\right)\tilde{h}_{vv}^{(0)} = -(2\pi i - \beta\omega)(1-\Omega^2)\left[2k\tilde{h}_{v\phi}^{(0)} + \omega\tilde{h}_{\phi\phi}^{(0)}\right].$$

At $(\omega, k) = \left(\frac{2\pi i}{\beta(1\mp\Omega)}, \pm\frac{2\pi i}{\beta(1\mp\Omega)}\right)$ both solutions are regular

$\Rightarrow$ $\lambda_\pm = \frac{2\pi}{\beta(1\mp\Omega\ell)}$ $\quad v_\pm = \mp 1$

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$$\Rightarrow \quad \lambda_\pm = \frac{2\pi}{\beta(1\mp\Omega\ell)} \qquad v_\pm = \mp 1$$

- Correlators of energy density from holography

$$\langle T^{\tau\tau}(\omega_E, k)T^{\tau\tau}(-\omega_E, -k)\rangle \propto \frac{\delta^2 S_{\text{ren.}}}{\delta\tilde{h}_{\tau\tau}^{(0)}\delta\tilde{h}_{\tau\tau}^{*(0)}} = \frac{k^2(4+k^2)}{2(\omega_E^2+k^2)}$$

  The pole skipping point is

$$(\omega, k) = \left(\frac{2\pi i}{\beta(1\mp\Omega)}, \pm\frac{2\pi i}{\beta(1\mp\Omega)}\right)$$

Pole skipping in CFT

- For CFT on cylinder

$$G_R(\omega,k)=\frac{c_L}{6}\left(\frac{2\pi}{\beta_L}\right)^3\left(\frac{2i}{\omega-k}+\pi\delta(\frac{k-\omega}{2})\right)\sinh\left[\frac{\beta_L k}{2}\right]\left|\Gamma\left(2+\frac{i\beta_L}{2\pi}k\right)\right|^2$$
$$-\frac{c_R}{6}\left(\frac{2\pi}{\beta_R}\right)^3\left(\frac{2i}{\omega+k}+\pi\delta(\frac{k+\omega}{2})\right)\sinh\left[\frac{\beta_R k}{2}\right]\left|\Gamma\left(2+\frac{i\beta_R}{2\pi}k\right)\right|^2.$$

  - From the first term $(\omega,k)=\left(\pm\frac{2\pi i}{\beta(1-\Omega)},\ \pm\frac{2\pi i}{\beta(1-\Omega)}\right)$
  - From the second term $(\omega,k)=\left(\pm\frac{2\pi i}{\beta(1+\Omega)},\ \mp\frac{2\pi i}{\beta(1+\Omega)}\right)$

[see also Haehl, Rozali, 1808.02898]

- Pole skipping is a generic feature of any CFT, including chaotic CFTs and non-chaotic CFTs.

Topologically Massive Gravity (TMG)

- A gravitational Chern-Simons deformation to Einstein gravity [Deser, Jackiw, Templeton, 1988]

$$S = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left(R + 2 + \frac{1}{2\mu}\varepsilon^{abc}\Gamma^d{}_{ae}\left(\partial_b\Gamma^e{}_{cd} + \frac{2}{3}\Gamma^e{}_{bf}\Gamma^f{}_{cd}\right)\right)$$

- Any solution of Einstein gravity is a solution of TMG

- Thermodynamics for rotating BTZ black holes [Krause, Larsen, hep-th/0508218]

$$M(\mu) = M + \frac{J}{\mu}, \qquad J(\mu) = J + \frac{M}{\mu}$$

- The angular direction and the decompactification limit

- The dual field theory for TMG on rotating BTZ is a CFT with $\beta_\pm = \beta(1 \mp \ell\Omega)$

  and $(c_L, c_R) = \frac{3\ell}{2G}\left(1 - \frac{1}{\mu}, 1 + \frac{1}{\mu}\right)$

- When $\mu\ell < 1$: negative central charge; Black hole instability [Park, hep-th/0608165]

- Chiral point $\mu\ell = 1$ [Li, Song, Strominger, 0801.4566]

Chaos in TMG from OTOC

- Profile for shock wave and OTOC ($\mu \neq 1$)

$$h(\phi) = c_1\, e^{-\frac{2\pi\phi}{\beta(1+\Omega)}} + c_2\, e^{\frac{2\pi\phi}{\beta(1-\Omega)}} + c_3\, e^{\frac{2\pi(\Omega-\mu)\phi}{\beta(1-\Omega^2)}}$$

$$\mathrm{OTOC}(t,\varphi) = 1 - \varepsilon\, e^{\frac{2\pi}{\beta}t} h(\Omega t - \varphi)$$

- Naively, we have three Lyapunov exponents (non-maximal chaos?)

$$\lambda_\pm = \frac{2\pi}{\beta(1\mp\Omega)}\,, \quad \lambda_m = \frac{2\pi(1-\mu\Omega)}{\beta(1-\Omega^2)} \qquad v_\pm = \pm 1\,, \quad v_m = \frac{1-\mu\Omega}{\Omega-\mu}$$

- Periodicity in $\phi : h(\phi) \to h(\phi \bmod 2\pi)$ there is a constraint equation among c_i

$$h(\phi) = \frac{1+\mu}{1-e^{-\frac{4\pi^2}{\beta(1+\Omega)}}}\, e^{-\frac{2\pi\phi}{\beta(1+\Omega)}} - \frac{1-\mu}{e^{\frac{4\pi^2}{\beta(1-\Omega)}}-1}\, e^{\frac{2\pi\phi}{\beta(1-\Omega)}} - \frac{2}{1-e^{\frac{4\pi^2(\Omega-\mu)}{\beta(1-\Omega^2)}}}\, e^{\frac{2\pi(\Omega-\mu)\phi}{\beta(1-\Omega^2)}}$$

- There is one independent "instantaneous Lyapunov exponent"

$$\lambda_{\mathrm{inst.}}(t) = \frac{2\pi}{\beta} + \frac{\partial_t h(\Omega t)}{h(\Omega t)}$$

High T limit of instantaneous Lyapunov exponents

- When $\mu > 1$

$$\lambda_{\text{inst.}} = \begin{cases} \lambda_- \,, & \text{if } \; t \in \left[0, \; \frac{\pi(1+\Omega)}{\Omega}\right) \\ \lambda_+ \,, & \text{if } \; t \in \left[\frac{\pi(1+\Omega)}{\Omega}, \; \frac{2\pi}{\Omega}\right) \end{cases}$$

$$\langle \lambda_{\text{inst.}} \rangle = \frac{2\pi}{\beta}$$

- When $\mu < 1$

$\Omega < \mu$

$$\lambda_{\text{inst.}} = \begin{cases} \lambda_m \,, & \text{if } \; t \in \left[0, \; \frac{2\pi(1+\Omega)}{\Omega(1+\mu)}\right) \\ \lambda_+ \,, & \text{if } \; t \in \left[\frac{2\pi(1+\Omega)}{\Omega(1+\mu)}, \; \frac{2\pi}{\Omega}\right) \end{cases} \qquad \langle \lambda_{\text{inst.}} \rangle = \frac{2\pi}{\beta}$$

$\mu < \Omega$

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Lyapunov exponent and butterfly velocities from OTOC

- In the high temperature limit and $|\Omega t-\varphi| \ll 1$
  - When $\mu > 1$

$$g(t,\varphi) = 1-\varepsilon_{VW}\begin{cases} (\mu-1)\, e^{\lambda_+(t-\varphi)}\,, & \text{if } \Omega t < \varphi \\ (1+\mu)\, e^{\lambda_-(t+\varphi)}\,, & \text{if } \Omega t \geq \varphi \end{cases}$$

  - When $\mu < 1$

$\Omega < \mu$

$$g(t,\varphi) = 1-\varepsilon_{VW}\begin{cases} -(1-\mu)\, e^{\lambda_+(t-\varphi)}\,, & \text{if } \Omega t < \varphi \\ -2\, e^{\lambda_m\left(t-\frac{\varphi}{v_m}\right)}\,, & \text{if } \Omega t \geq \varphi \end{cases}$$

violate the speed bound; $v_m > c$

$\mu < \Omega$

$$g(t,\varphi) = 1-\varepsilon_{VW}\begin{cases} 2\, e^{\lambda_m\left(t-\frac{\varphi}{v_m}\right)}\,, & \text{if } \Omega t < \varphi \\ (1+\mu)\, e^{\lambda_-(t+\varphi)}\,, & \text{if } \Omega t \geq \varphi \end{cases}$$

- The dual system is a non-maximally chaotic system

Chaos in TMG from OTOC

- When $\mu = 1$, the profile of the shock wave is

$$h(\phi) = \frac{\#}{4r_+}\left[\left(\frac{1+2r_+\phi}{1-e^{-\frac{4\pi^2}{\beta(1+\Omega)}}} + \frac{4\pi r_+ e^{\frac{4\pi^2}{\beta(1+\Omega)}}}{(e^{\frac{4\pi^2}{\beta(1+\Omega)}}-1)^2}\right)e^{\frac{-2\pi\phi}{\beta(1+\Omega)}} + \frac{1}{r_+}\frac{1}{e^{\frac{4\pi^2}{\beta(1-\Omega)}}-1}e^{\frac{2\pi\phi}{\beta(1-\Omega)}}\right]$$

- OTOC

$$g(t,\varphi) = 1 - \varepsilon_{VW}\begin{cases} \frac{\#}{4r_+^2}\, e^{\lambda_+(t-\varphi)}, & \text{if } \Omega t < \varphi \\ \frac{\#}{4r_+}\, e^{\lambda_-(t+\varphi)}, & \text{if } \Omega t > \varphi \end{cases}$$

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$$h(\phi) = \frac{\#}{4r_+}\left[\left(\frac{1+2r_+\phi}{1-e^{-\frac{4\pi^2}{\beta(1+\Omega)}}} + \frac{4\pi r_+ e^{\frac{4\pi^2}{\beta(1+\Omega)}}}{(e^{\frac{4\pi^2}{\beta(1+\Omega)}}-1)^2}\right)e^{\frac{-2\pi\phi}{\beta(1+\Omega)}} + \frac{1}{r_+}\frac{1}{e^{\frac{4\pi^2}{\beta(1-\Omega)}}-1}e^{\frac{2\pi\phi}{\beta(1-\Omega)}}\right]$$

- OTOC

$$g(t,\varphi) = 1 - \varepsilon_{VW}\begin{cases}\frac{\#}{4r_+^2}\; e^{\lambda_+(t-\varphi)}\,, & \text{if }\; \Omega t < \varphi\\ \frac{\#}{4r_+}\; e^{\lambda_-(t+\varphi)}\,, & \text{if }\; \Omega t > \varphi\end{cases}$$

- The system duals to rotating BTZ black hole in TMG is non-maximally chaotic

- MSS chaos bound is always saturated

- If we impose the velocity bound on the butterfly velocity, only $\mu \geq 1$ is allowed

Pole skipping from holography

- Pole skipping from near horizon EOM

$$e_{vv}^{(0)}h_{vv}^{(0)}+e_{vr}^{(0)}h_{vr}^{(0)}+e_{v\phi}^{(0)}h_{v\phi}^{(0)}+e_{r\phi}^{(0)}h_{r\phi}^{(0)}+e_{\phi\phi}^{(0)}h_{\phi\phi}^{(0)}+e_{vv}^{(1)}h_{vv}^{(1)}+e_{v\phi}^{(1)}h_{v\phi}^{(1)}=0$$

$$\left(\frac{2\pi i}{\beta(1\mp\Omega)}, \quad \mp\frac{2\pi i}{\beta(1\mp\Omega)}\right) \quad \& \quad \left(\frac{2\pi i(1-\Omega\mu)}{\beta(1-\Omega^2)}, \quad \frac{2i\pi(\Omega-\mu)}{\beta(1-\Omega^2)}\right)$$

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- Pole skipping from holographic massive mode

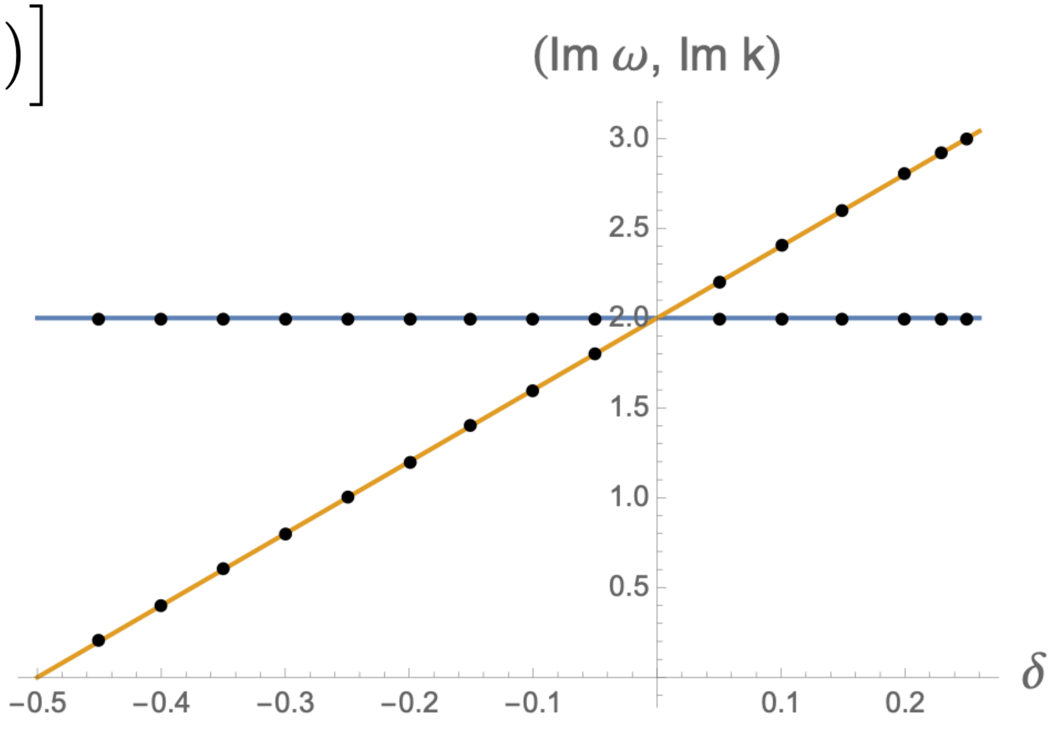
$$h_{ij}(\rho)=e^{-i\omega T+ikX}\Big[h_{ij}^{(0)}+\rho h_{ij}^{(1)}+\rho^2h_{ij}^{(2)}+\rho^{-\delta}\left(b_{ij}^{(0)}+\rho b_{ij}^{(1)}+\rho^2 b_{ij}^{(2)}+\cdots\right)$$
$$+\rho^{\delta+1}\left(c_{ij}^{(0)}+\rho c_{ij}^{(1)}+\rho^2 c_{ij}^{(2)}+\cdots\right)\Big]$$

$$\mu=2\delta+1$$

$$G_R^{t^{00}t^{00}}(\omega,k)\propto\frac{c_{tt}^{(0)}}{b_{tt}^{(0)}}$$

$$\boxed{\mathrm{Im}\omega=-\mathrm{Im}k+4(1+\delta)}$$

$$\boxed{\mathrm{Im}\omega=\mathrm{Im}k-4\delta}$$



Pole skipping from CFT

- The massive graviton is dual to an operator with conformal dimension $(2+\delta, \delta)$

- The retarded Green's function

$$G_R(\omega,k) \propto \sin\left[\delta + \frac{i\beta_R}{2\pi}\left(\frac{\omega-k}{2}\right)\right] \sin\left[2+\delta + \frac{i\beta_L}{2\pi}\left(\frac{\omega+k}{2}\right)\right] \times$$
$$\times \left|\Gamma\left(\delta + \frac{i\beta_R}{2\pi}\left(\frac{\omega-k}{2}\right)\right)\right|^2 \left|\Gamma\left(2+\delta + \frac{i\beta_L}{2\pi}\left(\frac{\omega+k}{2}\right)\right)\right|^2$$

- Pole-skipping point

$$(\omega,k) = \left(\frac{2\pi i(1-\Omega\mu)}{\beta(1-\Omega^2)},\ \frac{2i\pi(\Omega-\mu)}{\beta(1-\Omega^2)}\right)$$

Conclusion

- OTOC and pole-skipping (from near horizon dynamics, holographic correlators, CFT calculations) are two features of quantum chaos
- For rotating BTZ in 3D Einstein gravity, we find a match between the two methods in the high temperature limit
- For rotating BTZ in 3D TMG, we only find a “partial” match between these two methods in the high temperature limit
- For non maximally chaotic system, OTOC and pole-skipping seems to be two independent approaches

Conclusion

- OTOC and pole-skipping (from near horizon dynamics, holographic correlators, CFT calculations) are two features of quantum chaos

- For rotating BTZ in 3D Einstein gravity, we find a match between the two methods in the high temperature limit

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- It would be interesting to study other non-maximal chaotic systems (from CFT or holography) [Choi et al. 2010.08558]

Recent developments…

- pole-skipping is a generic phenomenon, for lots of correlators
- “Pole collision” can be used to define the equilibrium time scale, equilibrium length scale
- Hydrodynamical origin of chaos? Generic features of maximally chaotic systems? Perspectives of hydro EFT of chaos
- Non-maximally chaotic systems
- …

Thank you!